

Subject. (S.o.1)

Date. _____

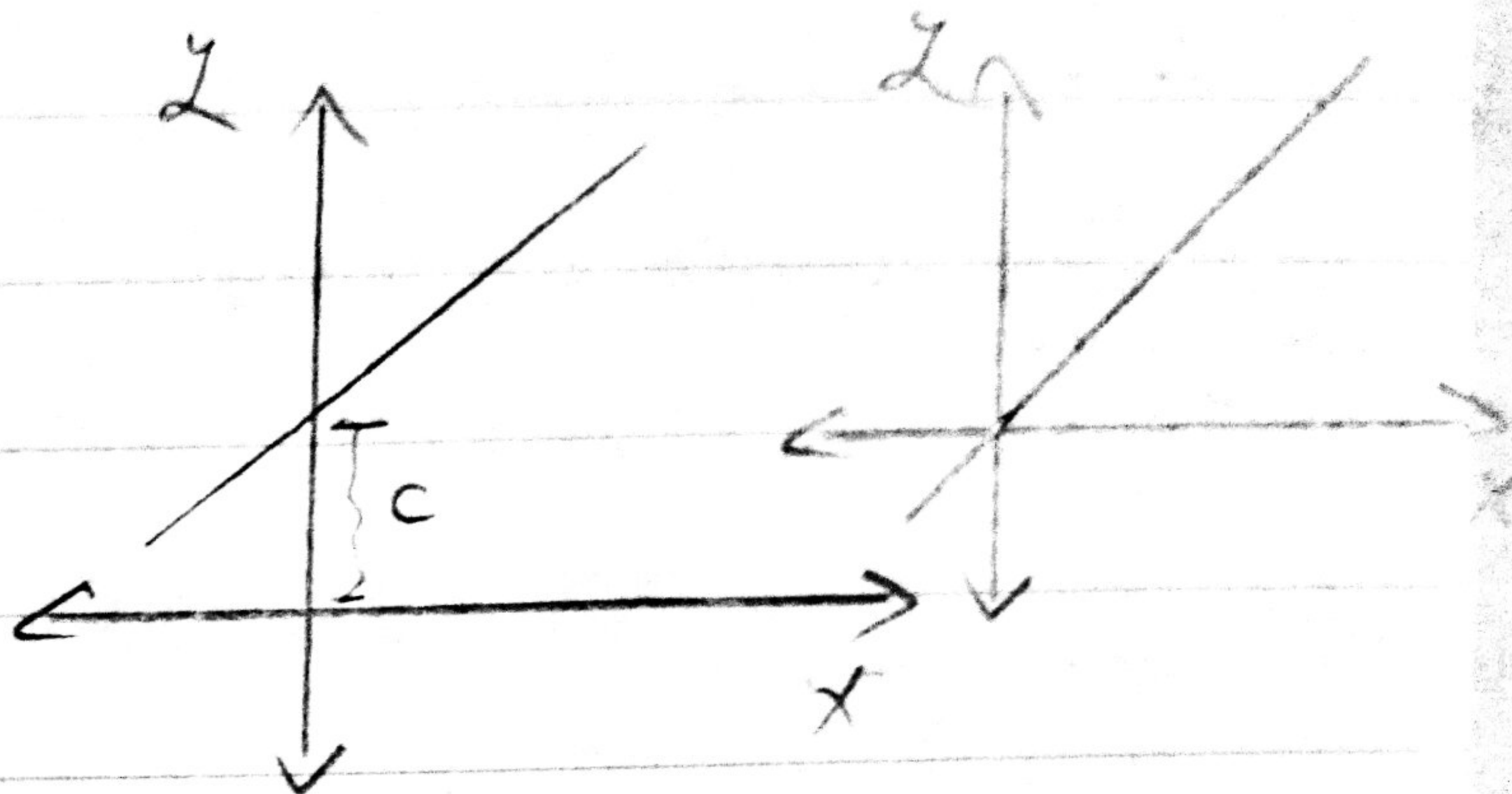
Two Fold eqn. of two Lines

$$y = mx$$

$$y = mx + c$$

Zero form:-

$$y - mx = 0, y - mx - c = 0$$



* Slope of straight line:-

$$m: \textcircled{1} \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = m$$

$$\textcircled{2} \tan \theta = m$$

$$\textcircled{3} \frac{dy}{dx} = m$$

Forcing Straight Line equation

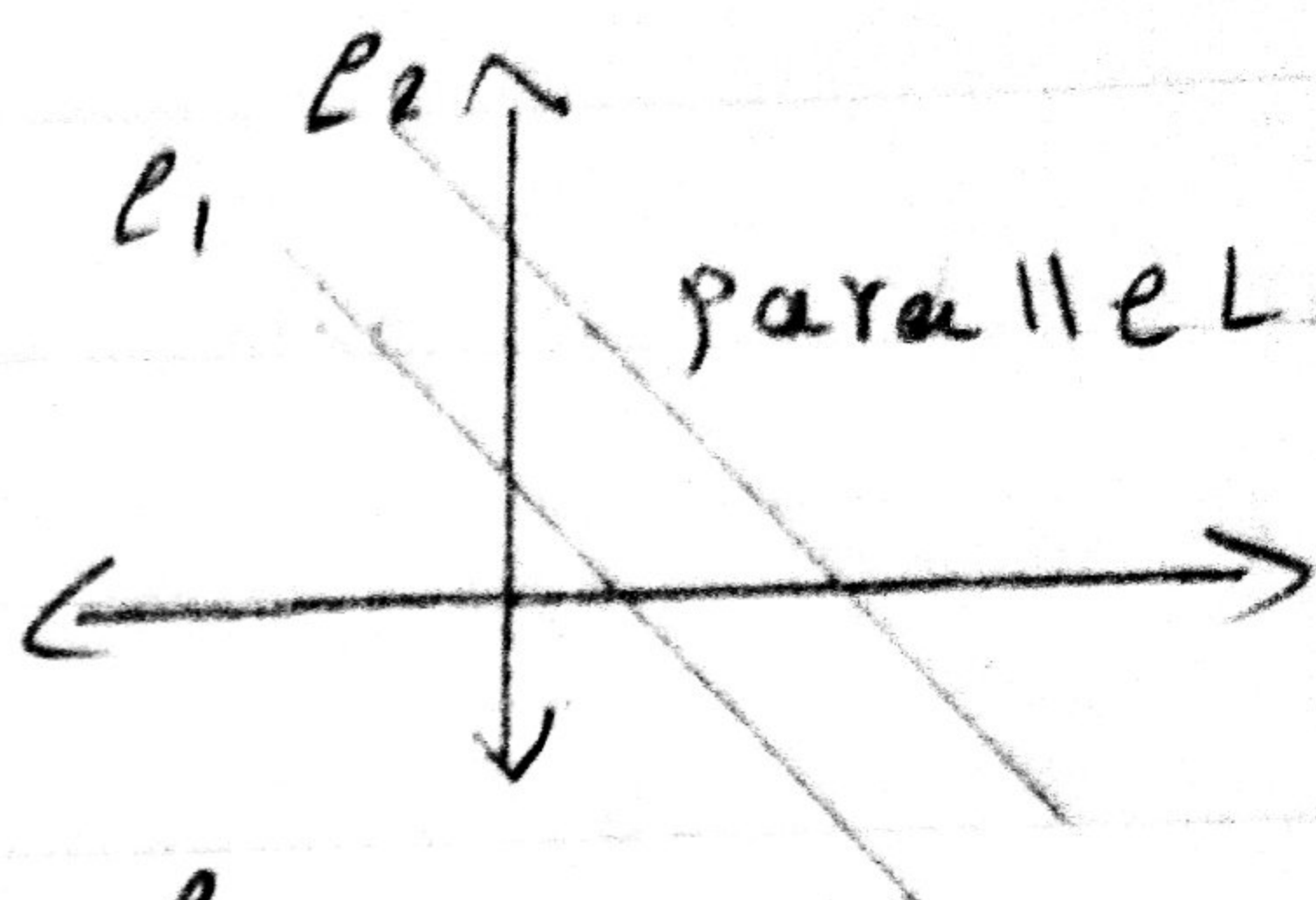
$$\textcircled{1} \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\textcircled{2} \frac{y - y_1}{x - x_1} = m$$

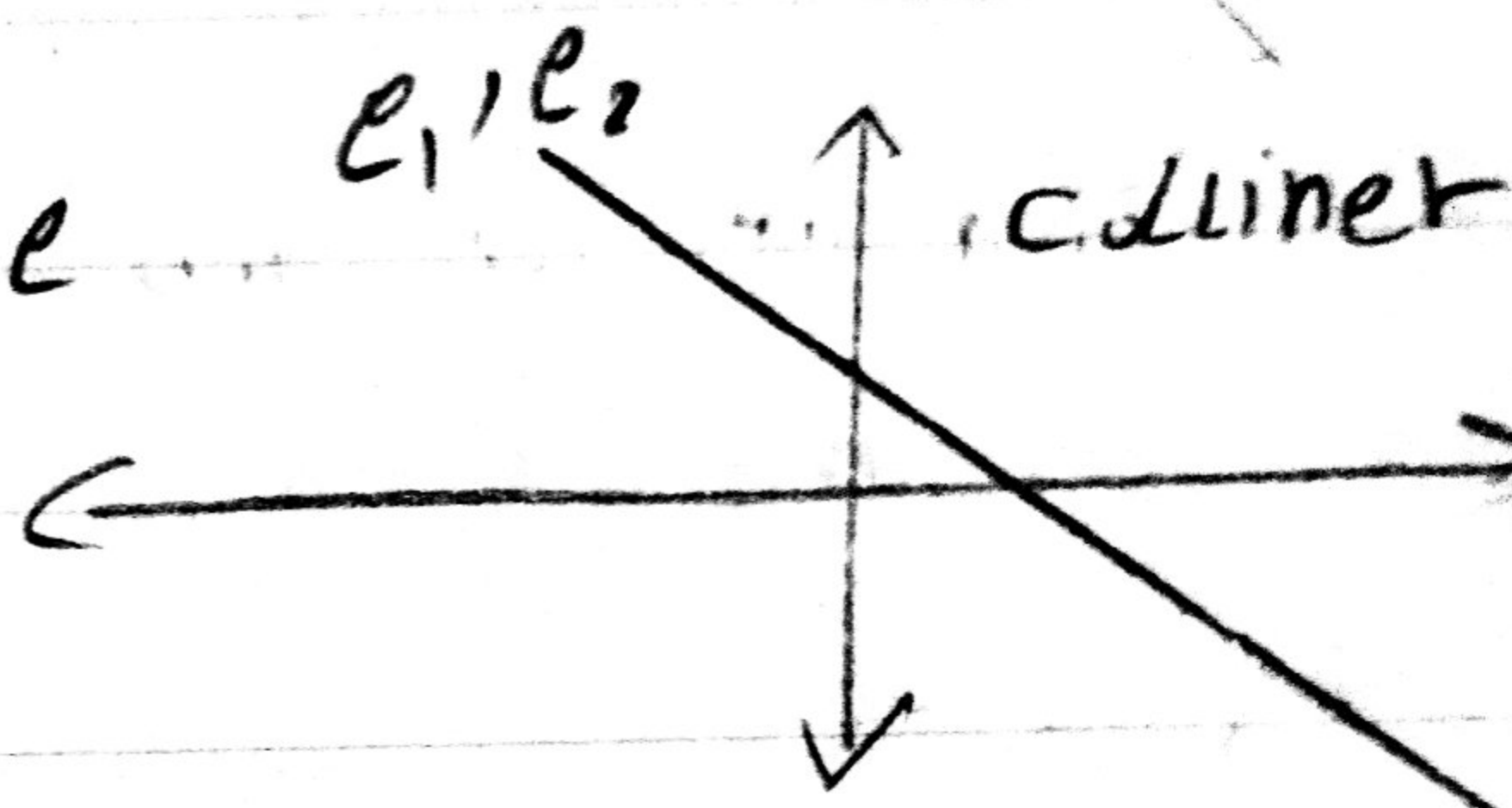
$$L_1: a_1x + b_1y + c_1 = 0$$

$$L_2: a_2x + b_2y + c_2 = 0$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow L_1 \parallel L_2$$



$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \text{The Same}$$



* Angle between two intersected lines :-

$$\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$\tan 90 = \frac{1}{0}$$

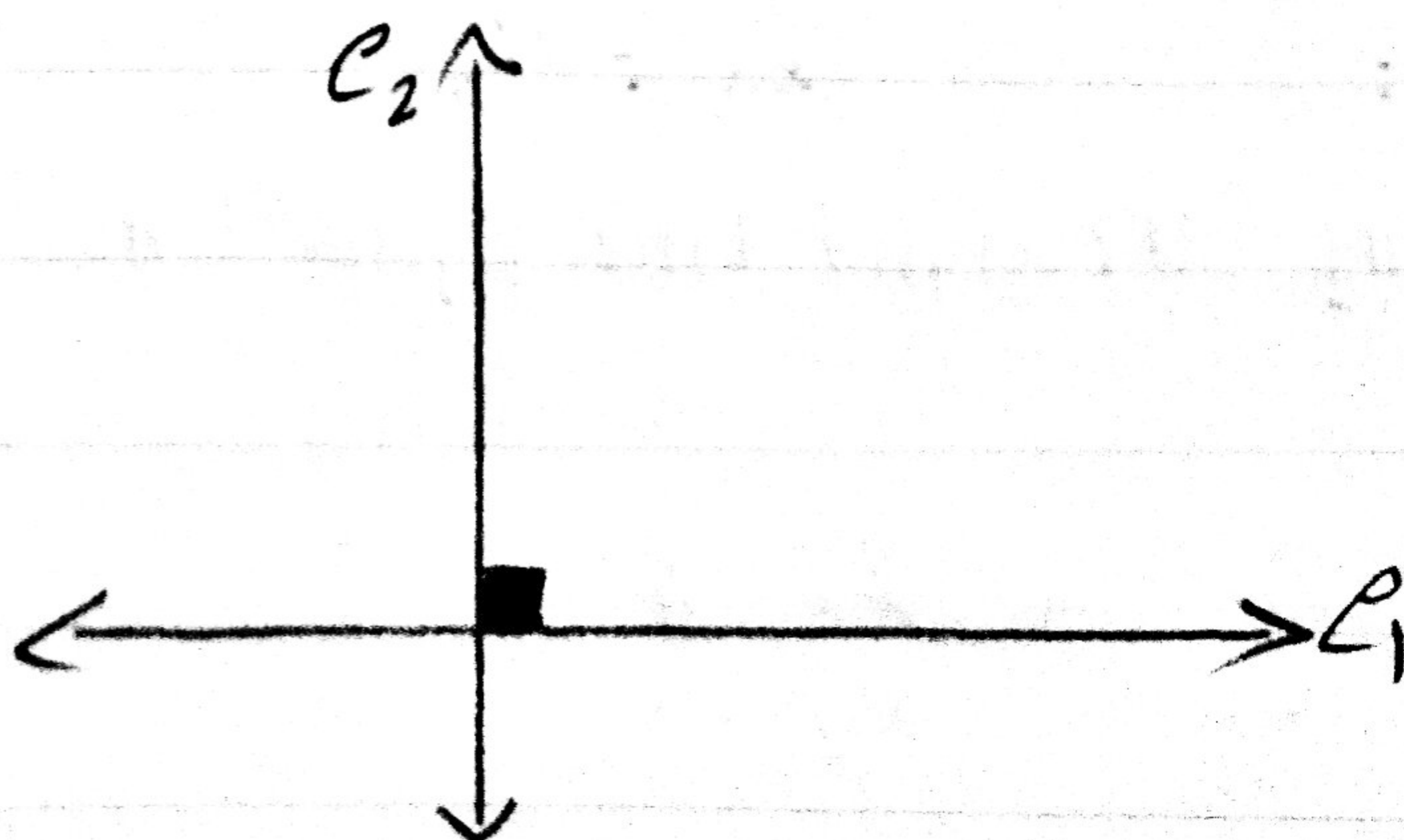
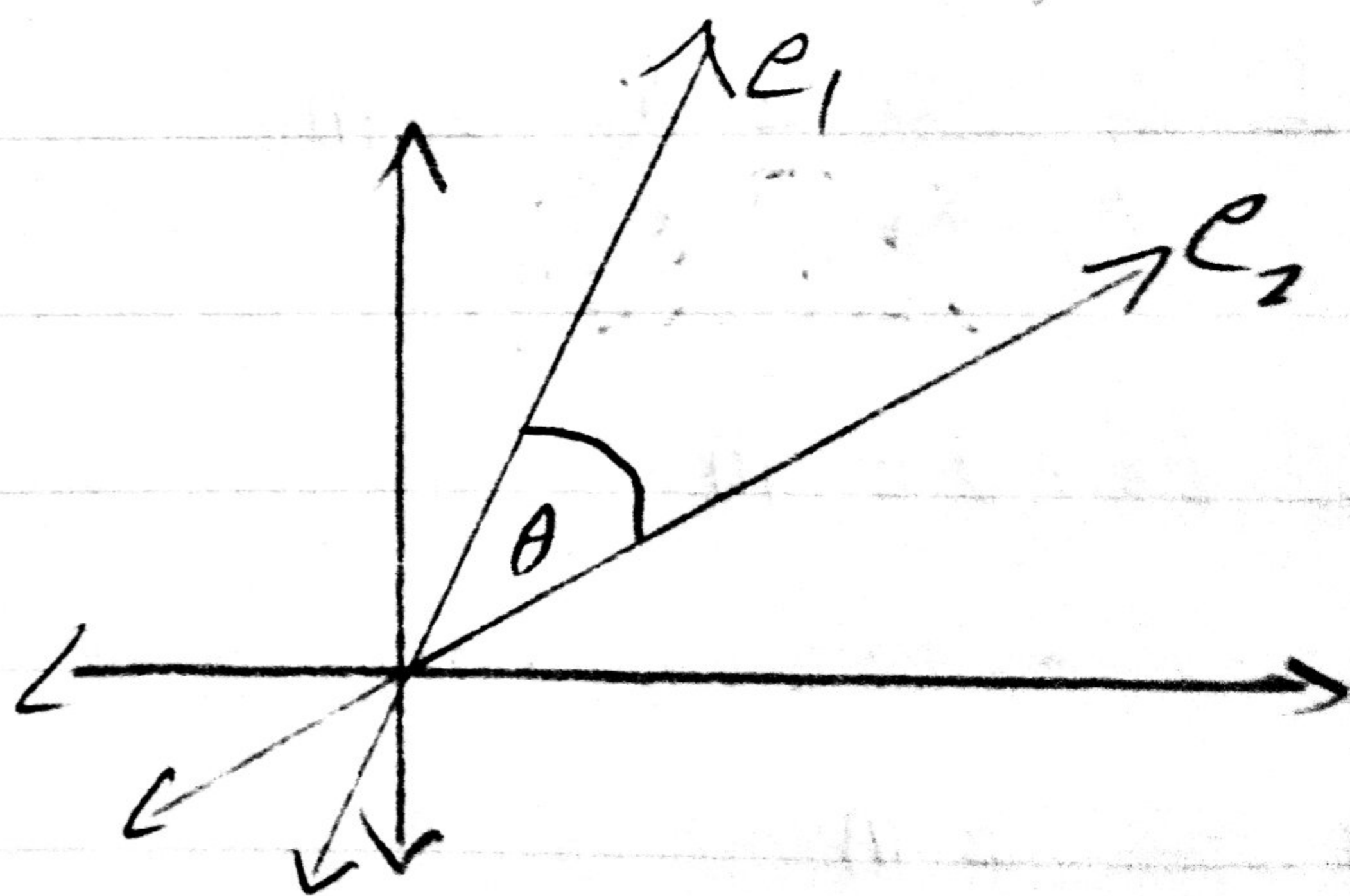
$$1 + m_1 m_2 = 0$$

$$m_1 m_2 = -1$$



$$y = 2x$$

$$y = -\frac{1}{2}x$$

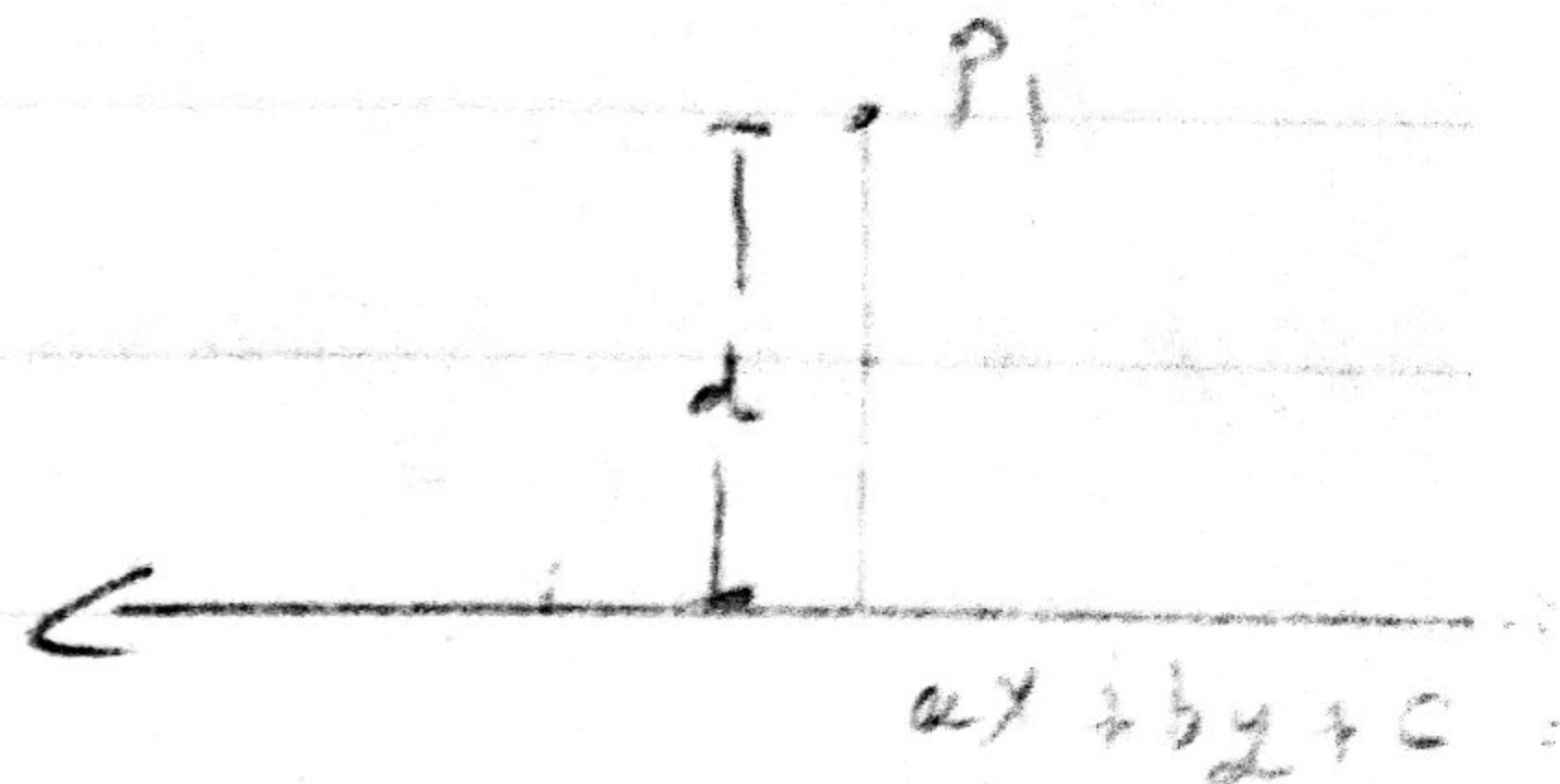


Subject. _____

Date. _____

Distance between $P_1(x_1, y_1)$ and a line:-

$$d = \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$$



Two fold eqn of Two lines passing Through The origin

$$ax^2 + by^2 + 2hxy = 0 \quad (\text{Homogenous of 2nd degree})$$

$$a + b\left(\frac{y}{x}\right)^2 + 2h\frac{y}{x} = 0$$

$$\frac{y}{x} = \frac{-2h \pm \sqrt{(2h)^2 - 4ab}}{2b}$$

$$\frac{y}{x} = \frac{-h \pm \sqrt{h^2 - ab}}{b} = m$$

$$\frac{y}{x} = m_1 \quad \frac{y}{x} = m_2$$

$h^2 - ab \rightarrow 0 \rightarrow \text{Collinear}$
 $\rightarrow < 0 \rightarrow \text{نقاط متممة}$
 $\rightarrow > 0 \rightarrow \text{مقاطع}$

Angle between two straight Lines:-

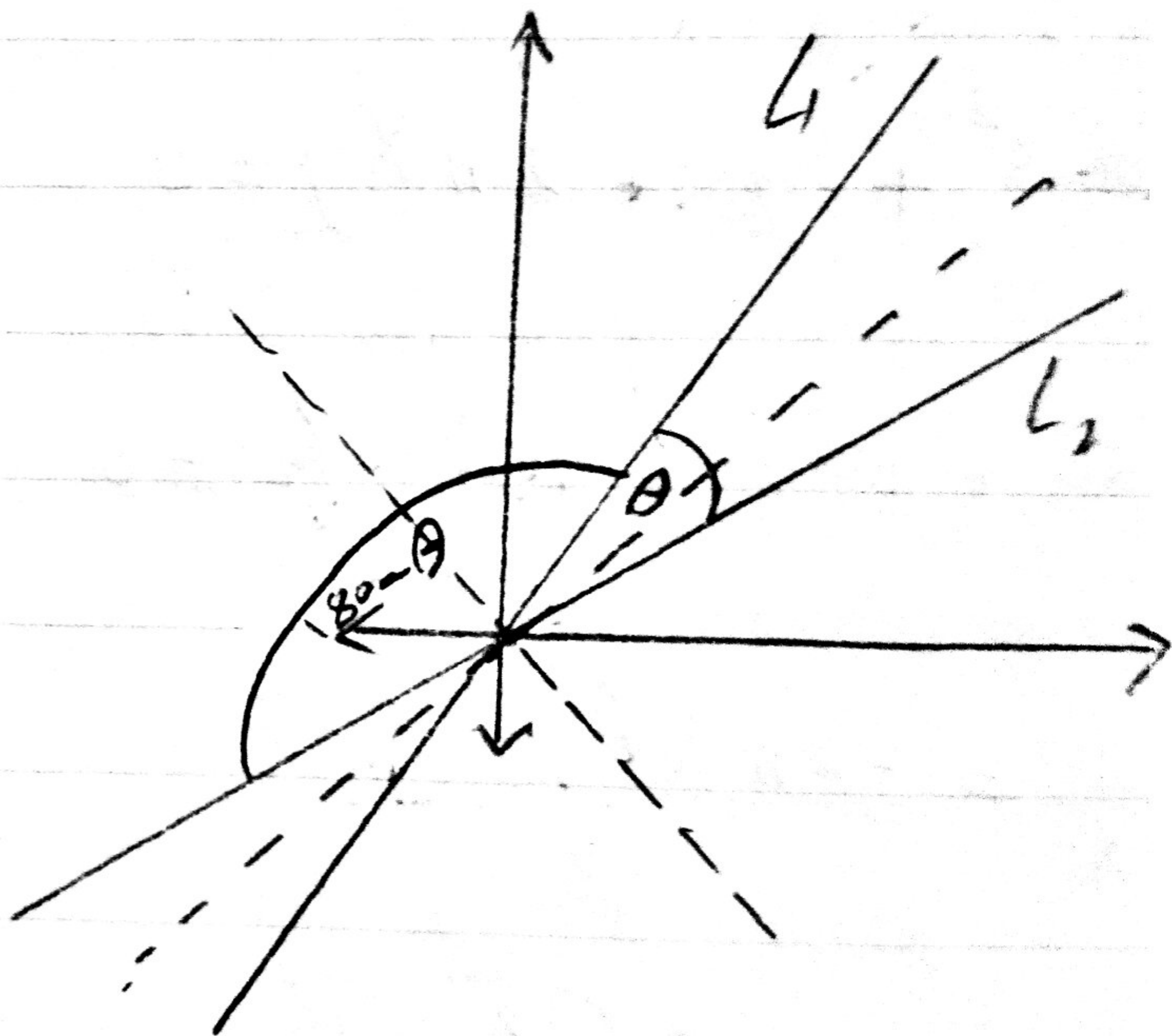
$$\tan \theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

Two fold eqn. of Two Bisectors to angle between two straight Lines passing through the origin:-

$$\frac{x^2 - y^2}{a-b} = \frac{xy}{h}$$

$$B_1 \perp B_2$$

perpendicular



Ex:-

prove that the following eqn represent two lines eqns.

and find these lines, The Angle between them,

and the two fold eqn of the bisectors:-

$$3x^2 - xy - 10y^2 = 0$$

Sol \hookrightarrow

$$ax^2 + by^2 + 2hxy = 0$$

$$a = 3, b = -10, h = -\frac{1}{2}$$

$$h^2 - ab = \frac{1}{4} - (3 \times -10) = 30.25 \quad \#$$

$$\frac{y}{x} = \frac{-h \pm \sqrt{h^2 - ab}}{b}$$

$$\frac{y}{x} = \frac{-3}{5} \quad \frac{y}{x} = \frac{1}{2} \quad \#$$

$$\tan \theta = \frac{2\sqrt{h^2 - ab}}{a + b} = \checkmark$$

$$\theta = \tan^{-1} \checkmark = -57.7 \quad \#$$

$$\frac{x^2 - y^2}{3 + 10} = \frac{xy}{-\frac{1}{2}}$$

$$x^2 + 26xy - y^2 = 0 \quad \#$$

Ex:-

prove that the two fold equation of the two lines passing through $(0,0)$ and make Angle α with

$$X - y = 0 \text{ is } X^2 + 2 \sec(2\alpha) Xy + y^2 = 0$$

$$X - y = 0 \rightarrow B_1$$

$$X + y = 0 \rightarrow B_2$$

$$(X^2 - y^2) = 0$$

$$\frac{X^2 - y^2}{a - b} = \frac{Xy}{h}$$

$$\frac{1}{\frac{1}{a-b}} = \frac{0}{\frac{1}{h}}$$

$$a - b = 0 \Rightarrow a = b$$

$$\tan \theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

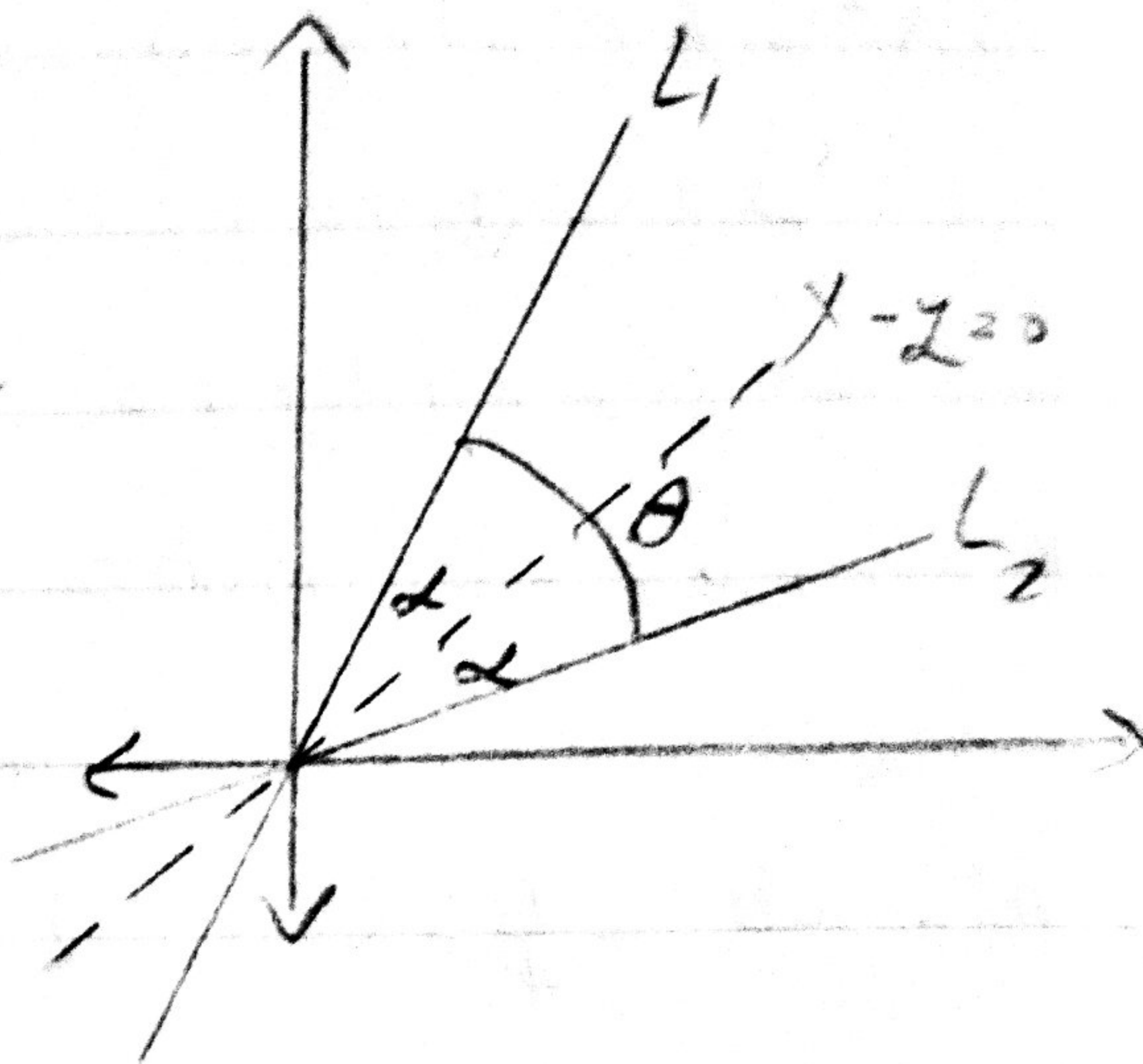
$$\tan \theta = \frac{2\sqrt{h^2 - a^2}}{2a}$$

$$a \tan \theta = \sqrt{h^2 - a^2}$$

$$a^2 \tan^2 \theta = h^2 - a^2$$

$$h^2 = a^2 (1 + \tan^2 \theta)$$

$$h^2 = a^2 \sec^2 2\alpha$$



$$h = \pm a \sec 2\alpha$$

$$aX^2 + by^2 + 2hXy = 0$$

$$aX^2 + a'y^2 + 2a \sec 2\alpha Xy = 0$$

$$X^2 + 2 \sec 2\alpha Xy + y^2 = 0$$